Instructions: Complete each of the following exercises for practice.

1. Verify that the Divergence Theorem holds by computing $\iiint_R \text{div}(\mathbf{F})\ dV$ and $\iint_{\partial S} \mathbf{F} \cdot d\mathbf{S}$ separately.
   (a) $\mathbf{F}(x,y,z) = \langle 3x, xy, 2xz\rangle$, R the cube bounded by the planes $x=0$, $y=0$, $z=0$, $x=1$, $y=1$, and $z=1$
   (b) $\mathbf{F}(x,y,z) = \langle y^2z^3, 2yz, 4z^2\rangle$, R the solid enclosed by $z = x^2+y^2$ and $z=9$
   (c) $\mathbf{F}(x,y,z) = \langle z, y, x\rangle$, R the solid ball $x^2+y^2+z^2 \le 16$
   (d) $\mathbf{F}(x,y,z) = \langle x^2, -y, z\rangle$, R the solid cylinder $y^2+z^2 \le 9$, $0 \le x \le 2$

2. Use the Divergence Theorem to compute the flux of $\mathbf{F}$ across S.
   (a) $\mathbf{F}(x,y,z) = \langle xye^z, xy^2z^3, -ye^z\rangle$, S the surface of the box bounded by the coordinate planes and $x=3$, $y=2$, and $z=1$
   (b) $\mathbf{F}(x,y,z) = \langle x^2yz, xy^2z, xyz^2\rangle$, S the surface of the box bounded by the coordinate planes and $x=a$, $y=b$, and $z=c$ for $a,b,c>0$
   (c) $\mathbf{F}(x,y,z) = \langle 3xy^2, xe^z, z^3\rangle$, S the boundary of the solid cylinder $y^2+z^2 \le 1$, $-1 \le x \le 2$
   (d) $\mathbf{F}(x,y,z) = \langle x^3+y^3, y^3+z^3, z^3+x^3\rangle$, S the sphere of radius 2 centered at the origin
   (e) $\mathbf{F}(x,y,z) = \langle xe^y, z-e^y, -xy\rangle$, S the ellipsoid $x^2+2y^2+3z^2=4$
   (f) $\mathbf{F}(x,y,z) = \langle z, y, zx\rangle$, S the boundary of the tetrahedron enclosed by the coordinate planes and the plane $\dfrac{x}{a}+\dfrac{y}{b}+\dfrac{z}{c}=1$ where $a,b,c>0$
   (g) $\mathbf{F}(x,y,z) = \langle 2x^3+y^3, y^3+z^3, 3y^2z\rangle$, S the boundary of the region enclosed by paraboloid $z = 1-x^2-y^2$ and the xy-plane
   (h) $\mathbf{F}(x,y,z) = \langle xy+2xz, x^2+y^2, xy-z^2\rangle$, S the boundary of the region in the cylinder $x^2+y^2=4$ and between planes $z=y-2$ and $z=0$

3. Suppose $R \subseteq \mathbb{R}^3$ satisfies the conditions of the divergence Theorem, $\mathbf{C}$ is a constant vector, and assume vector field $\mathbf{F}$ and scalar fields f and g have continuous second-order partial derivatives on an open region containing R. Prove each of the following.
   (a) $\iint_{\partial R} \mathbf{C} \cdot d\mathbf{S} = 0$
   (b) $Vol(R) = \frac{1}{3}\iint_{\partial R} \langle x,y,z\rangle \cdot d\mathbf{S}$
   (c) $\iint_{\partial R} \text{curl}(\mathbf{F}) \cdot d\mathbf{S} = 0$
   (d) $\iint_{\partial R} D_{\mathbf{n}} f\ dS = \iiint_R \nabla^2 f\ dV$
   (e) $\iint_{\partial R} f\nabla g \cdot d\mathbf{S} = \iiint_R (f\nabla^2 g + \nabla f \cdot \nabla g)\ dV$
   (f) $\iint_{\partial R} (f\nabla g - g\nabla f) \cdot d\mathbf{S} = \iiint_R (f\nabla^2 g - g\nabla^2 f)\ dV$